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THE THREE-WAY FACTOR ANALYSIS OF SPATIO-TEMPORAL CHANGE OF POLAND'S SOCIO-ECONOMIC STRUCTURE

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ABSTRACT. The three-way model of factor analysis is introduced into the study of the evolution of spatial patterns. This model achieves an integration of the temporal and spatial approaches to socio-economic structure. Three-way factor analysis is used in an empirical study of Poland's regional structure in the years 1961–1970.

Introduction

This study contains an application of factor analysis to the study of spatio-temporal variation in Poland's socio-economic structure. A crucial element in the reasoning of this method of factor analysis is the inclusion of both the spatial and temporal dimensions of variation.

The goal of this study is principally to try out the applied method and to ascertain its investigatory potential, i.e. its methodological aspect, since the material analysed embraces only a ten-year time series of the variables under investigation. This is too short a period for obtaining results that would form a basis for a complete typology and generalization of the variation process under consideration. The limitation of the time-period of the analysis followed from difficulties that arose in the complication of documentary evidence containing data characterizing Poland's socio-economic structure in both spatial and temporal dimensions.

In the study of changes in the spatial patterns of socio-economic phenomena, progress until the present has been and still is inadequate, and lags behind the study of of socio-economic development. The questions of socio-economic development form the central theme of research both from a philosophical point of view, especially that of dialectical and historical materialism, as well as from the viewpoint of the theory and practice of the socio-economic sciences. Development is, according to dialectical materialism, the most important determinant defining reality.

However, the assumptions and methodology of the study of spatial patterns have, above all, put emphasis on the synchronic approach, one referring to the relations between coexistent entities, to the detriment of the diachronic approach which operates on the time axis, and aims to capture the processes of change in these patterns.

The synchronic approach has found expression in investigations into spatial patterns in the form of multifarious structural models embracing the study of simultaneous relations, i.e. containing an assumption about the stability of the dependence relations between the variables of a model. Various models of interaction, of networks, flows, etc. belong of this type.

Without going into any more closely reasoned appraisal of this type of model, it should be stated that they have a limited scope of application, for the dependences established in them refer only to a certain segment of time during which. The permanence of these dependences is observed. This limits considerably their explanatory and prognostic function, and reduces their role to a description of the existing spatial structure.

The diachronic approach relates to the study of the temporal variation of phenomena and is based on an analysis of time series. The point is to discover the process of change with an indication of its mechanism which constitutes the basic link in the development of knowledge about socio-economic phenomena. The relinquishment of the study of temporal changes not only makes it impossible to know the past but also the present and the future, for only knowledge of the development process enables one to determine and characterize the permanence of a structure and the trends in its transformation. The study of the evolution of spatial patterns does not have to consider the genesis of these patterns in depth, and in particular does not have to reconstruct them in the form of a historical description, but should concentrate on the changes relating to the elements that are induced by the covariation of conditioning influences and lead to a transformation of the patterns.

The study of the evolution of spatial patterns of socio-economic phenomena encounters methodological problems, for it requires the combination of both synchronic and the diachronic approaches; the spatial structure of phenomena and their temporal variation are studied together. Not only should these two different approaches complement each other, but they should also accomplish their spatio-temporal integration in an overall, holistic manner.

The factor analysis of spatio-temporal variation should be included in the group of models achieving such an integration, in addition to others such as stochastic models of the evolution of spatial patterns based on Markov chains and stochastic simulation. Although it rests on different assumptions, it resolves the conflict between synchrony and diachrony in the study of spatial patterns and yields a combination of both approaches and not a simple sum of these.

The model

The application of the factor-analytic model to the study of spatio-temporal variation in a structure is based on the assumption that a socio-economic structure is a whole defined on a set of characteristics which describe it exhaustively. Any integrated temporal and spatial approach to this whole is equivalent to the category of socio-economic time-space continuum. The concept of the time-space continuum serves in the reconstruction of the variation in the socio-economic structure conditioned by the development processes that govern it. One of the methods of holistic description of this time-space continuum is three-way factor analysis in its broader meaning [1].This method identifies the latent structure of time-space continuum, which can be identified with the essential structure, i.e. a structure representing the intrinsic elements of variation in the patterns under investigation. The factors represent the intrinsic spatial subpatterns of phenomena, or partial spaces complemented by the temporal dimension.

[1] The name "factor analysis" is ambiguous and in its more general meaning it comprises a number mathematical and statistical methods. Among these the best-known and widely applied are the factor analysis in its narrower sense and the principal components analysis.

These subpatterns do not exhaust the whole population of partial spaces, but they are of fundamental importance both in terms of the relative importance of the phenomena represented, and in terms of temporal variation. To conclude, we can identify them as "subpatterns-change carries" in the formation of the internal structure of the overall space.

The study of socio-economic time-space continuum requires the handling of a three-mode data matrix which can be represented in the shape of a parallelepiped of information containing socio-economic events classified in three dimensions: spatial location: M, characteristic: N, and time of occurrence: T (Fig. 1).

The three-way factor-analysis model has the form:

$$x_{nmt} = \sum_{s} \sum_{p} \sum_{q} a_{ns} b_{mp} c_{tq} g_{spq}$$

It is assumed in this model that there exist latent dimensions S, P, Q lyining at the base of the observational dimensions N, M, T, the former being represented by factors of the three-mode matrix of observations X_{nmt}. The intrinsic factors define the essential structure of the time-space continuum under study (Fig. 2). The coefficients a_{ns}, b_{mp}, c_{tq} are elements of the matrix of factor coefficients A [n, s], B [m, p], C [t, q] and describe the elements of the observational dimensions in terms of latent dimensions. The g_{spq} coefficients are elements

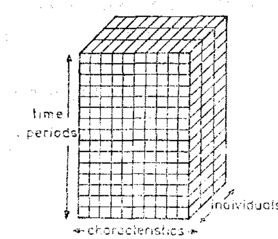

Fig. 1

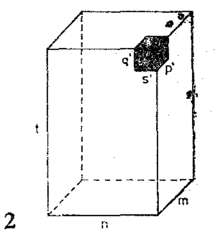

Fig. 2

It should be noted at this point that the classical factor model is applied in the analysis of two-mode data matrices (see Czyż 1971). For example: a set of towns is characterized in terms of their functions, and the correlation matrix for the functions of towns is factor-analysed. The factors identify the intrinsic functions, i.e. those having an appreciable share in the overall variation.

In the study of the socio-economic time-space continuum, the factor model must undergo extension to the three-way model [2].

of the matrix of factor scores G [s, p, q], called the core matrix.

In the matrix of observations X [n, m, t] each element represents the score of a phenomenon for a certain combination of categories of the observational dimensions. Each element of the core matrix, on the other hand, G [s, p, q] is the score of an event in latent dimensions. The reduced core matrix G [s′, p′, q′] is interpreted as the essential structure matrix describing the intrinsic relations present in the pattern under investigation.

[2] The first attempt at formulating a three-way model of factor analysis was undertaken by Tucker (1963) in psychology. New proposals in the strategy of factor analysis applications in geography have been formulated by Chojnicki and Czyż (1976).

The application of three-way factor-analytic model is a substantial advance in the analysis of spatio-temporal variation. Working with the traditional two-way factor-analytic model examining spatial patterns, we can only make comparisons within a discontinuous series of various spatial structures, which are of an autonomous character, because the factors are derived from different temporal structures[3]. On the other hand, such a comparison does not allow us to determine the variation such factors in time, since they are differently constructed. Hence, such a model is analogous to comparative statics models (cf. Harvey 1967).

It is possible, however, to reconstruct the process of change from the spatial structure by using the time-space continuum factor analytic model. Its elements are handled as dependent variables of other elements of the structure in a time series. The factor structure is not invariable in such an approach, but it undergoes change together with the change of relations. The moment the number of new relations is too great, there comes about a change of the structure itself.

An application

In order to verify the effectiveness of the spatio-temporal model using the factor-analytic approach, empirical studies were conducted on the variation of Poland's regional structure in the years 1961–1970[4].

For this purpose, the following hypothesis has been formulated: The factors of temporal variation of events in Poland's regional structure include factors representing, with respect to their spatial distribution, two differing levels of variation and correspond to factors working throughout the pattern or factors characteristic for groups of elements or subpatterns, with change being of a continuous character or possibly occurring only in certain subperiods of time.

TABLE 1. SOCIO-ECONOMIC VARIABLES

Variable number	Variable name
1	Population size
2	Total industrial output (zl)
3	Electricity generation (kWh)
4	Industrial employment
5	Purchase of agricultural products (zl)
6	Total investment outlays (zl)
7	Employment in the service sector
8	Retail sales (zl)
9	Household consumption of electricity (kWh)
10	Habitable rooms completed

The three-way factor-analytic model is introduced as a method of testing this hypothesis. The reference system is constituted by the administrative division of Poland into 17 voivodships. The characteristics introduced into the information matrix concern production factors (population, employment, investments), production in industry and agriculture, service activities and the level of consumption (Tab. 1). It should be noted, however, that these characteristics do not represent all, the properties of Poland's socio-economic space, which may affect the contents of the results. The time series is short, for it refers to a period of ten years (1961–1970), which consequently limits the possibility of interpreting the results correctly with regard to their merits.

The factorial solution was reached by the principal components method[5]. The algorithm of the three-way principal components analysis is as shown in Figure 3.

[3] An approach of this kind was employed by Chojnicki and Czyż (1972) in their research on the variation of Poland's regional structure.

[4] Czyż (1971), Chojnicki and Czyż (1972, 1974) introduced factor analysis into the study of Poland's regional economic structure.

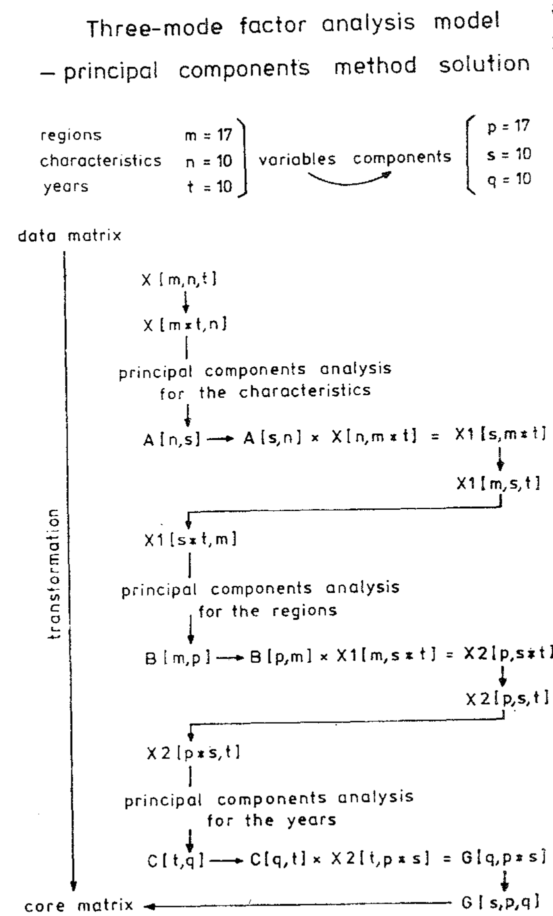

Fig. 3

(1) The extraction of the principal components is carried out in a conventional manner for the two-mode matrix X [m. t, n], which designates an orthogonal transformation of the set of variables n into a set s of new variables, called components. The examination of the characteristic roots and the vector coefficients of the components allows us to state that the first two components account for 90% of the total variance of the original variables (Tab. 2A). The first component (y_{1s}) absorbs 79% of the variance and is associated with 9 of the original variables. It can be interpreted as representing non-agricultural socio-economic development. The second component (y_{2s}), whose participation in the total variance amounts to 11%, is identified with agricultural production growth.

(2) Next, principal components analysis is applied successively for the X [s.t,m] and the X [p,s,t] types of two-mode matrices. An analysis of the results allows one to identify four intrinsic regional subpatterns corresponding to the first four components (Tab. 2B). The first subpattern, corresponding to component y_{1p} accounting for 64% of the total variance includes Białystok, Koszalin, and Olsztyn voivodships; the second subpattern, which concentrates 15% of variance, is Warsaw voivodship; the third subpattern consists of Gdańsk, Katowice, Kraków, Łódź, and Wrocław voivodships (10% of variance), the fourth subpattern is formed by Cracow, Opole, Poznań, and Wrocław voivodships (7% of variance). Each of the isolated regional subpatterns can be interpreted as a group of regions which change similarly through time and they account jointly for 96% of the spatial variance of events. An analysis of the coefficients for q components for the years leads to the statement that variation in the regional structure proceeds in a twofold manner: firstly it is of a continuous character, 86% of total variance falls to changes of this type; and secondly, its changes are of a short-term character (13% of total variance).

[5] The programme and computations were carried out on an "Odra 1204" computer by Dr. Z. Piasecki. The principal components have been extracted from the correlation matrix. See Morrison (1967).

TABLE 2. PRINCIPAL COMPONENTS COEFFICIENTS*

A.

Variables	Components y_{1s}	y_{2s}
1	0.333	0.117
2	0.339	−0.215
3	0.261	−0.257
4	0.322	−0.299
5	0.109	0.852
6	0.346	−0.028
7	0.031	0.180
8	0.351	0.120
9	0.350	−0.054
10	0.340	0.090
λ	7.902	1.138
Accumulated percentage of total variance	79.02	90.40

B.

Voivodships	Components y_{1p}	y_{2p}	y_{3p}	y_{4p}
Białystok	0.300	−0.059	0.014	0.015
Bydgoszcz	0.059	−0.597	−0.054	0.005
Gdańsk	0.177	−0.065	0.497	−0.357
Katowice	−0.258	0.060	0.363	−0.064
Kielce	0.274	−0.135	0.190	−0.154
Koszalin	0.301	−0.135	0.067	0.015
Cracow	−0.187	−0.003	0.417	0.447
Lublin	0.187	0.131	−0.162	0.057
Łódź	−0.181	−0.466	0.345	−0.380
Olsztyn	0.300	−0.265	−0.016	0.057
Opole	0.265	−0.100	0.168	0.320
Poznań	−0.151	0.089	−0.005	0.413
Rzeszów	0.269	−0.448	0.250	0.194
Szczecin	0.292	−0.004	0.165	0.077
Warsaw	−0.247	0.308	0.137	−0.202
Wrocław	−0.228	−0.058	0.309	0.369
Zielona Góra	0.295	0.063	0.153	0.011
λ	10.838	2.567	1.729	1.213
Accumulated percentage of total variance	63.7	78.8	89.0	96.2

C.

Years	Components y_{1q}	y_{2q}
1961	0.297	−0.424
1962	0.310	−0.356
1963	0.315	−0.323
1964	0.328	−0.222
1965	0.337	−0.075
1966	0.338	0.045
1967	0.331	0.189
1968	0.316	0.319
1969	0.298	0.418
1970	0.285	0.468
λ	8.601	1.303
Accumulated percentage of total variance	86.01	99.04

* Only those components have been considered which correspond to eigenvalues greater than one.

These two components of change on the time dimension are interpreted as a continuum of changes in the years 1961–1970 (component y_{1q}) and as changes in the 1968–1970 subperiod designating a certain phase of socio-economic development (component y_{2q}) (Tab. 2C).

(3) Following the extraction of the principal components for the set of variables, regions and time periods, we can calculate the component scores for the three dimensions and write them in the form of the core matrix G [s, p, q]. In this way the initial set X is transformed from the space with observational dimensions N, M, T into the set G in the space with latent dimensions S, P, Q. Subsequently because the first few components accounted for considerable proportion of total variance, matrix G [s, p, q] is reduced in dimension from G [10, 17, 10] to G [2, 4, 2]. The loss of information when passing to the reduced component space is negligible and amount to 10%, 4% and 1% respectively for the

dimensions S, P, Q. The reduced core matrix is identified with essential structure matrix.

The three-mode reduced core matrix G [2, 4, 2] can be written in the form of two two-mode matrices: they contain the intrinsic component scores of socio-economic events for the components of regional subpatterns in the two periods of change: 1961–1970 and 1968–1970 (Tab. 3). Each element of a core matrix so written is a standarized score on component y_{1s} or y_{2s} for

TABLE 3. REDUCED CORE MATRIX

(a) Changes in the years 1961–1970 (y_{1q})

y_p \ y_s	y_{1s} Non-agricultural socio-economic development	y_{2s} Growth of agricultural production
y_{1p} Białystok, Koszalin, Olsztyn	−0.156	0.032
y_{2p} Warsaw	0.372	0.002
y_{3p} Gdańsk, Katowice, Cracow, Łódź, Wrocław	−0.176	−0.095
y_{4p} Cracow, Opole, Poznań, Wrocław	−0.323	−0.071

(b) Changes in the years 1968–1970 (y_{2q})

y_p \ y_s	y_{1s} Non-agricultural socio-economic development	y_{2s} Growth of agricultural production
y_{1p} Białystok, Koszalin, Olsztyn	0.286	0.419
y_{2p} Warsaw	−0.637	−1.026
y_{3p} Gdańsk, Katowice, Cracow, Łódź, Wrocław	0.492	0.047
y_{4p} Cracow, Opole, Poznań, Wrocław	0.310	0.302

regional patterns (y_{1p}, y_{2p}, y_{3p}, y_{4p}) with the mean equal to zero for the period of change under investigation.

The core matrix scores may be interpreted in the following way. In the years 1961–1970 (y_{1q}) the non-agricultural component (y_{1s}) in the socio-economic development for regional subpatterns varies between −0.32 and +0.37. Only the Warsaw voivodship subpattern (y_{2p}) reaches a score exceeding the mean in this period of change, while the subpattern of agricultural voivodships (y_{1p}) is characterized by scores below the mean for this time-period. The scores on the agricultural component (y_{2s}) are underdifferentiated and they are found in all regional subpatterns almost on the mean level for this period of change. In the subperiod 1968–1970 (y_{2q}) the distribution of the y_{1s} component scores is reversed: the lowest scores, below the mean for this subperiod, occur in Warsaw voivodship and the remaining subpatterns have their scores above the mean (0.29; 0.31; 0.49). The scores on the agricultural growth component (y_{2s}) remain more sharply differentiated. The score for Warsaw voivodship is much below the mean for this subperiod.

Conclusions

It should be stated in conclusion that the empirical studies were chiefly of an experimental character, on account of the need to check the three-way factor-analytic model. However, the results of these studies have borne out the preliminary hypothesis concerning the essential structure of Poland's socio-economic time-space continuum and have disclosed some aspects of its form.

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